

Greatest Common Divisor (GCD)

Boise Math Circle

- **GCD:** The Greatest Common Divisor of two numbers is the greatest number which divides (goes into) both numbers..
- **Division:** Dividing numbers can reduce the work needed to find a GCD. For example to find the GCD of 167 and 72, we first divide them: “167 divided by 72 is equal to 2 with a remainder of 23.” Put another way,
$$167 = 2 \times 72 + 23$$
Finding the GCD of 167 and 72 now reduces to finding the GCD of 72 and 23. That’s because any divisor of 167 and 72 must also divide 72 and 23, and vice versa.
- **Euclidean Algorithm:** We can find the GCD by repeating the above thinking. To begin, divide the first number by the second number. Next the second number becomes the new first number, and the remainder becomes the new second number, and divide again. Repeat until you get a remainder of 0. The GCD is the last nonzero remainder.
- **Example:** Find the GCD of **167** and **72**.
(fill in the missing values)

$$\boxed{167} = \underline{\quad} \times \boxed{72} + \boxed{23}$$

$$\boxed{72} = \underline{\quad} \times \boxed{23} + \boxed{3}$$

$$\boxed{23} = \underline{\quad} \times \boxed{3} + \boxed{2}$$

$$\boxed{3} = \underline{\quad} \times \boxed{\quad} + \boxed{\quad}$$

$$\boxed{\quad} = \underline{\quad} \times \boxed{\quad} + \boxed{\quad}$$

The GCD of 167 and 72 is ____.

- Some questions:
 - What GCD value is the most common?
 - How likely is it compared with other possible GCD values?

Let’s find out. The answers may surprise you!