

# The Math of Rearranging People

## Task 1: Sorting People

In a group of 4 or 5, stand in one of the orders below.

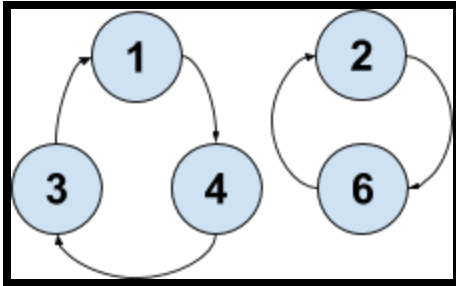
- Random order
- By age, from youngest to oldest
- By the number of letters in your last name, from least to greatest
- By height, from shortest to tallest
- By birth date, by time after January 1
- By foot size, from smallest to largest

Then, rearrange into one of the OTHER orders above, using these rules:

Rule	Starting Order	Ending Order	Number of "Moves"
Use only the operation of swapping two people.			
Use only double swaps, that is, whenever you swap one pair you have to swap another.			
Use only rotation (someone on the end moves to the other end).			
Use only swapping two people who are next to each other.			
Use only rotation, or swapping the left-most two people.			

# Drawing Permutations

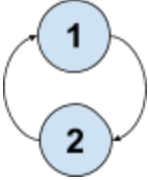
For today, we'll write down a permutation using "cycle graphs":



- Arrows show how the numbers move.
- The smallest number in a "cycle" is at the top.
- Any numbers that are not shown do not move.

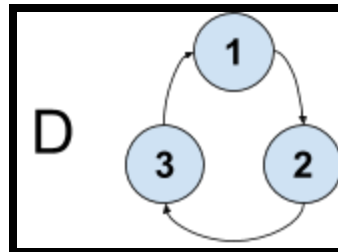
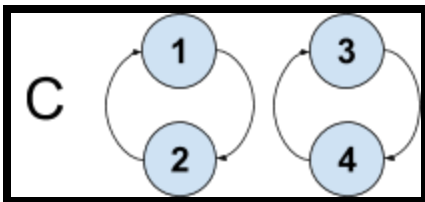
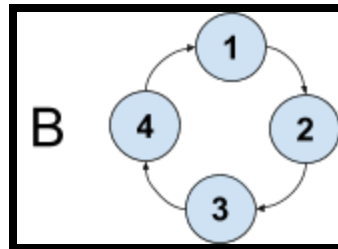
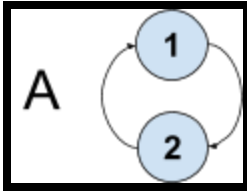
← In this permutation, 1 goes to 4, which goes to 3, which goes to 1. Meanwhile, 2 switches places with 6, and no other numbers move.

Task 2: Draw all the permutations of 1, 2, 3, 4.

<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">(no switching)</div>			

## Task 3: Algebra of Permutations

Consider the following permutations of 1, 2, 3, 4:



An “inverse” of a permutation is a permutation that “undoes” it.

1. Draw the inverse of A.
2. Draw the inverse of B.
3. Draw the inverse of C.
4. Draw the inverse of D.

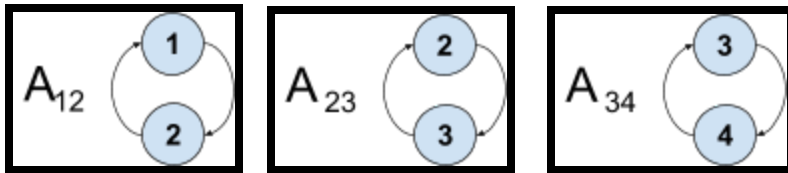
The “product” of two permutations is the permutation caused by doing the first permutation, then the second. (*Note*: it matters which happens first.)

1. Draw the product  $A \times B$  (first A, then B).
2. Draw the product  $A \times C$ .

3. Draw the product  $C \times A$ .
4. Draw the product  $A \times B \times C$ .
5. Draw the product  $D \times B$ .
6. Draw the product  $D \times B \times D$ .
7. Draw the product  $D \times B \times (\text{the inverse of } B)$ .

## Task 4: Elementary Swaps

The following permutations are called the “elementary swaps”.



What permutations can you make using products of the elementary swaps?

## Task 5: Double Swaps

Call E, F, and G the three “double-swaps”. Draw them below:

What are all the different permutations you can create using products of only E, F, and G?  
(hint: try things like  $E \times G$ , or  $E \times F \times F \times G$ )