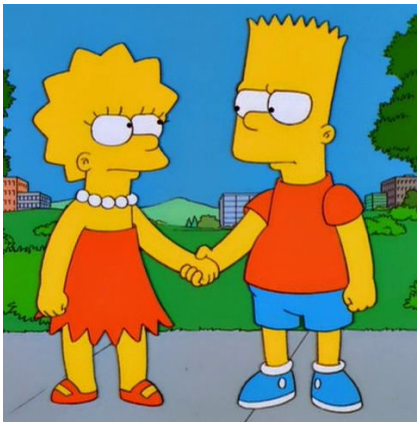
Bart vs. Lisa vs. Fractions



The Simpsons is a long-running animated series about a boy named Bart, his younger sister, Lisa, their family, and their town. One episode in the 14th season featured an unexpected situation:

- Bart, who usually didn't try hard in school, was told he would need to repeat 3rd grade.
- Lisa, who always tried hard in school, was told she would be skipping 2nd grade.
- Suddenly, the two siblings turned into rivals at school.

In this activity, we'll be thinking about hypothetical school-related problems to see whether Bart or Lisa comes out ahead.

Problem 1: Bart vs. Lisa vs. Spelling

- On Day 1, the teacher asked Lisa to spell "deductible", "harass", "recommend", and "splendiferous". The only one she knew was "deductible" (Lisa loves doing her taxes).
- On Day 2, the teacher asked Lisa to spell "handkerchief". She remembered this one was tricky, but got it right!
- On Day 1, the teacher asked Bart to spell "mathematics". He was stumped!
- On Day 2, the teachers asked Bart to spell "shorts", "harass", "horse", and "skateboard". To everyone's surprise, he spelled all the words except "horse" correctly!

Who got a bigger FRACTION of their spelling words right on Day 1?

Who got a bigger FRACTION of their spelling words right on Day 2?

Who got a bigger FRACTION of their spelling words right overall, (combining both days)? How do you think Bart and Lisa felt when they found out?

Problem 2: Bart vs. Lisa vs. Bouncing Balls

- On Day 1, Lisa bounced her ball 110 times in 410 seconds.
- On Day 2, Lisa bounced her ball 45 times in 140 seconds.
- On Day 1, Bart bounced his ball 12 times in 48 seconds.
- On Day 2, Bart bounced his ball 180 times in 580 seconds.

Who bounced the ball faster on Day 1?

Who bounced the ball faster on Day 2?

Overall (combining both days), who bounced the ball faster?

Problem 3: Bart vs. Lisa vs. Popularity Contest

- On Day 1, 15 of the 20 students Bart asked said they liked Lisa
- On Day 2, 50 of the 210 students Bart asked said they liked Lisa
- On Day 1, 40 of the 60 students Lisa asked said they liked Bart
- On Day 2, 20 of the 100 students Lisa asked said they liked Bart

Who was more popular on Day 1?

Who was more popular on Day 2?

Overall (combining both days), who was more popular?

Problem 4: Bart vs. Lisa vs. _____

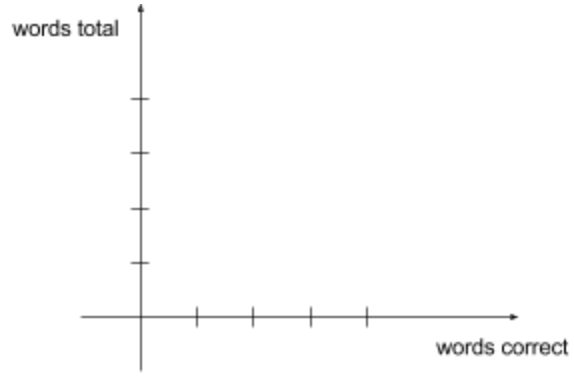
Try inventing a scenario with a similar pattern to the previous problems.

Graphing the data

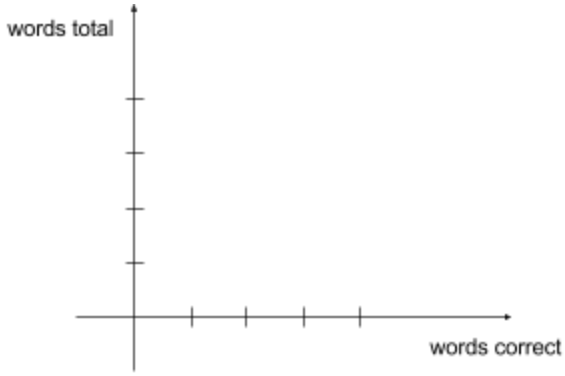
To get a better understanding of what is going on, let's return to Problem 1.

- On Day 1, Lisa got 1 out of 4 words correct. Let's write this as a **vector** (1,4).
- On Day 2, Lisa got 1 out of 1 word correct. Let's write this as a vector (1,1).
- On Day 1, Bart got 0 out of 1 word correct. Let's write this as a vector (0,1).
- On Day 2, Bart got 3 out of 4 words correct. Let's write this as a vector (3,4).

Plot Bart and Lisa's vector for Day 1



Plot Bart and Lisa's vectors for Day 2

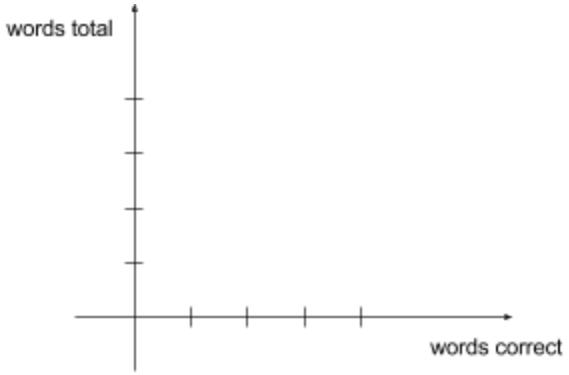


Remember that a vector is an **arrow** from the origin (0,0) to the coordinate.
In each case, how can you tell from the picture who got the bigger fraction of words right?

Now let's calculate the overall percentages.

- On Day 1 and Day 2 together, Lisa got 2 out of 5 words correct. We can write this using the vector sum (1,4)+(1,1)=(2,5).
- On Day 1 and Day 2 together, Bart got 3 out of 5 words correct. We can write this using the vector sum (0,1)+(3,4)=(3,5).

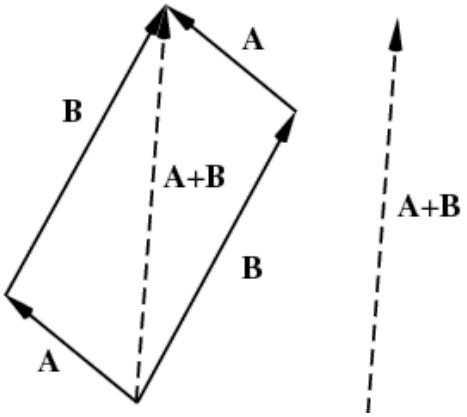
Plot Bart and Lisa's cumulative vectors.



How can you tell from the picture who got the bigger fraction of words right??

More about vectors

When you calculate a cumulative percentage, you are adding two vectors such as (1,4)+(1,1) to get (2,5), geometrically you join the two vectors from tip to tail:



Use this to help reason and answer the following questions.

Why is the cumulative fraction always **in between** the Day 1 and Day 2 fractions?

If I tell you the Day 1 fraction and the Day 2 fraction, does that determine the cumulative fraction? If not, what else do you need to know?

If I tell you the Day 1 fraction and the Day 2 fraction , can you fix the vectors so that for the cumulative fraction is anything you want in between?

What has to be true about the Day 1 and Day 2 fractions for Bart and Lisa so that a paradoxical situation is possible?

What has to be true about the Day 1 and Day 2 fraction for Bart and Lisa so that a paradoxical situation is impossible?