Here are the first few Fibonacci numbers:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$F(n)$	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610

n	16	17	18	19	20	21	22	23	24	25
$F(n)$	987	1,597	2,584	4,181	6,765	10,946	17,711	28,657	46,368	75,025

n	26	27	28	29	30
$F(n)$	121,393	196,418	317,811	514,229	832,040

Question 1. Which Fibonacci numbers are even? Which ones are odd? Why?

Question 2. Which Fibonacci numbers are divisible by 3?

Question 3. Use the table below to summarize what you found for divisibility by 2 and by 3. Is every 5th Fibonacci number divisible by 4?

Every ?'th Fibonacci number	Is divisible by ?

Question 4. Which Fibonacci numbers are divisible by 5?

Question 5. What patterns do you notice?
[Hint: Try focusing just on divisibility by 2, 3, 5, 8, etc]

Now try it with another sequence: $2^n - 1$. Here are the first few of these numbers:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$S(n)$	1	3	7	15	31	63	127	255	511	1023	2047	4095	8191	16383

n	15	16	17	18	19	20	21	22
$S(n)$	32767	65535	131071	262143	524287	1048575	2097151	4194303

Question 6. Which of these numbers are divisible by 3?

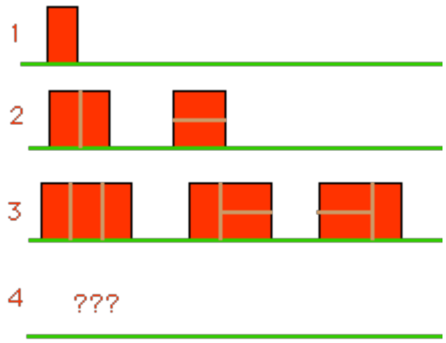
Question 7. Which are divisible by 7?

Question 8. Which are divisible by 15?

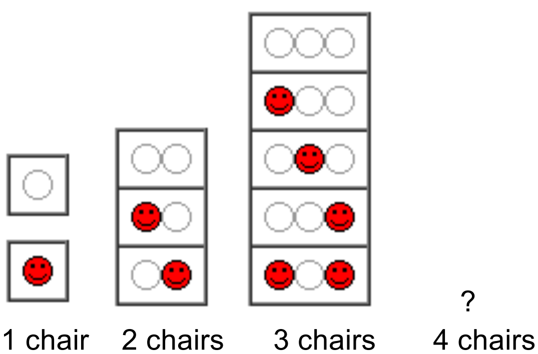
Question 9. How does this pattern compare with the patterns in the Fibonacci numbers? Can you explain why the pattern works?

Extensions

1. We have some bricks that are 1 unit by 2 units. We want to make a wall 2 units high, and n units wide. How many ways can it be done? The bricks can be vertical or horizontal.



2. Now you have a row of n chairs and you want to put students in some of the chairs. The students are taking an exam so they are not allowed to sit right next to each other. How many ways are there to do this?



3. Look at the "running totals" of the Fibonacci numbers. Here are the first few:

1+1 = 2
1+1+2 = 4
1+1+2+3 = 7
1+1+2+3+5 = 12
...

Do you see any patterns in this new sequence? Can you explain it?

4. What if you only add up every *other* Fibonacci number

1+2 =
1+2+5 =
1+2+5+13 =
1+2+5+13+34 =
...

What do you get? Can you explain this pattern?

5. Look at the "diagonals" in Pascal's triangle:

1 1 1 1 2 **1** 1 **3** 3 1 **1** 4 6 4 1 1 5 10 10 5 1 sum: 5	1 1 1 1 2 1 1 3 **3** 1 1 **4** 6 4 1 **1** 5 10 10 5 1 sum: 8	...

Can you explain this pattern?