

## Deranged Arrangements on the Range: Cards Out Of Order

Two cowboys, Dusty and Lefty, are sitting by the campfire after a long day on the range. Dusty says to Lefty, "Tomorrow we have to go visit our ten cattle feeding stations." "Just like every day," Lefty replies. "Every single day we visit those ten stations. We ride to station 1, then station 2, then station 3... we go on the same paths, in the same order. I'm sick of it." "Well then," says Dusty, "let's change the order."

You will need:

- The ace and 2-10 from a deck of cards. The ace will represent "1".

1. How many different orders can Dusty and Lefty visit those 10 cattle feeding stations?

"That's all well and good," says Lefty, "but it's not just changing the order. We're always at the same station at the same time of day. I'm tired of having lunch at station 5. I'd like to see what they're like at other times of day. Don't get me wrong, the sunrise at station 1 is pretty, but I wonder what it looks like from some of the other stations." Dusty says, "I see what you mean! Let's visit all the stations. But we'll make sure that we visit all the stations at a different time than usual."

2. How many ways can Dusty and Lefty visit the 10 cattle feeding stations, with their first stop being anywhere except station #1; their second stop being anywhere except station #2; ...; their tenth stop being anywhere except station #10? Every  $n$ 'th stop can be any station except the  $n$ 'th station.

*Terminology: a "permutation" is an arrangement of some items in some order. A "derangement" is a permutation where each  $n$ 'th item is not in the  $n$ 'th spot.*

Dusty and Lefty feel pretty happy with their new plan for the next day's work. After dinner, Lefty pulls out his trusty 10-note harmonica, "Ol' Squeaky," and starts playing some mournful tunes. "Good grief," groans Dusty. "You just keep playing the same notes, up and down, up and down. My ears are sick to their stomach." "What do you mean?" asks Lefty. "I'm just playing this mournful tune:"

10    1        3        2        8        6        7        5        9        4

"Sure," says Dusty, "There's a run of 5 notes going up: you're playing notes 1, 3, 6, 7, 9. That's not even the only one—there's 1, 2, 6, 7, 9, too. And if that wasn't bad enough, there's a couple runs of 5 notes going down: 10, 8, 7, 5, 4 is one, 10, 8, 6, 5, 4 is another." "Oh, I see," says Lefty, "the notes aren't consecutive, but going from left to right they all go up, or all go down. Well, if you don't like it then I won't play any more notes up and down. I reckon I can come up with a tune that you might like better."

3. Help Lefty arrange a tune with 10 notes. Better make sure that there are no runs going up, or runs going down, even non-consecutively. Can you help Lefty find a tune with no runs of 5 notes going up and no runs of 5 notes going down?
  
  
  
  
  
  
  
  
  
  
4. "Not good enough," says Dusty. "I don't even want to hear 4 up or 4 down. Plus note 10 is sounding mighty squeaky tonight." Help Lefty arrange a tune of just 9 notes with no runs of 4 notes going up and no runs of 4 notes going down.
  
  
  
  
  
  
  
  
  
  
5. Lefty is feeling pretty good about that last tune, and he feels like playing all the notes on his harmonica, no matter how squeaky they are. Lefty would like to find a tune of 10 notes with no runs of 4 notes going up and no runs of 4 notes going down. Is it possible?

Further resources:

- [http://www.numberphile.com/videos/order\\_chaos1.html](http://www.numberphile.com/videos/order_chaos1.html)
- [http://www.numberphile.com/videos/order\\_chaos2.html](http://www.numberphile.com/videos/order_chaos2.html)