Introduction to graph theory

We said last time that **combinatorics** is on the one hand the study of enumerations and counting, while on the other hand the study of structure and designs. This time we will get acquainted with the structures.

A **graph** is a diagram with dots, called **vertices** (singular: vertex), and lines joining the dots, called **edges**. A graph can represent relationships between different concepts.

A scheduling problem. Suppose you have 6 classes with students enrolled as follows.

Art	**Basketry**	**Collage**	**Dance**	**Economics**	**French**
Gabby	Hiram	Mira	Quincy	Phillipa	Jian
Hiram	Mira	Nano	Rana	Shilo	Uri
Jian	Nano	Qunicy	Shilo	Uri	Vela
Kala	Phillipa		Tensor		
Lonny					

If two classes have a common student, they cannot be offered at the same time. Draw a graph where:

- The vertices are the classes
- Two vertices are joined by an edge if the classes share a student

How many vertices are there? How many edges?

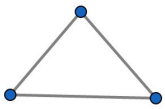
Counting and graphs

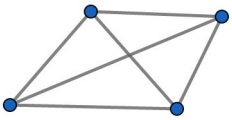
Consider each of the graphs below. Then fill in the table. Feel free to add your own graphs to the page!

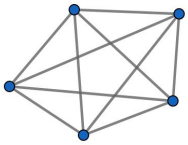
What patterns do you notice?

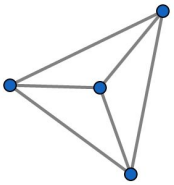
Graph	Number of vertices	Number of edges	Number of triangles

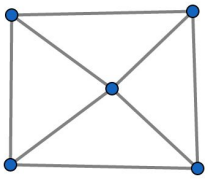
A:

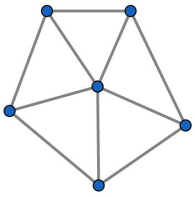
B: 

C: 

D: 

E: 

F: 

G: 

Degree sequences

The **degree** of a vertex is the number of edges that touch the vertex.

The **degree sequence** of a graph is the list of degrees of the vertices, ordered highest to lowest, with repetition. For example, the degree sequence of the graph to the right is (,,,).

Can you draw a graph with each of the degree sequences?

- (3,3,3,3)

- (2,2,2,2,2,1,1)

- (5,4,3,2,1)

- My secret made up graph:

- The degree sequence of my secret made up graph:

- Challenge your neighbor to make a graph from your degree sequence!

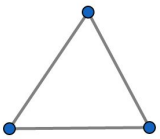
Independent sets

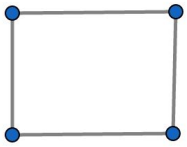
Redraw the scheduling graph from Page 1:

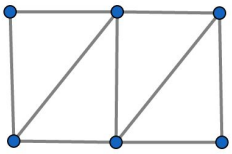
What is the largest number of courses that can be offered at one time?

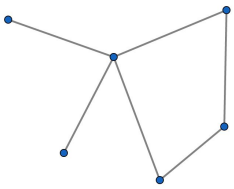
A set of vertices is called an **independent set** if there are no edges joining any of them. The **independence number** of a graph is the size of the largest independent set.

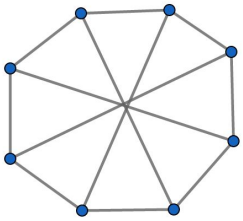
Find the independence number of each of the following graphs. Be sure to add your own!

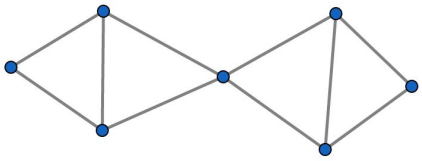
A: 

B: 

C: 

D: 

E: 

F: 

Can you create a graph with 30 vertices and independence number equal to 10?

Graph coloring

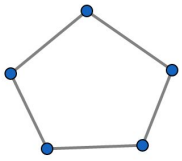
Redraw the scheduling graph from page 1:

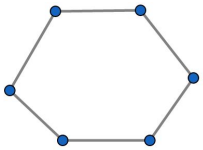
How many time slots do you need in order to run all the courses?

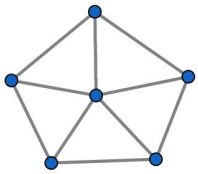
Stop! One way to find the answer to the above question is with **graph coloring**: Color the vertices of the graph in such a way that no edge has the same color at both ends. The number of time slots needed is the same as the number of colors needed!

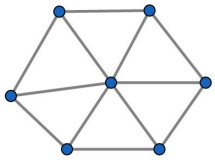
The **chromatic number** of a graph is the minimum number of colors you need to make a graph coloring.

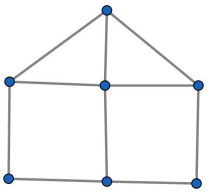
What is the chromatic number of each of the following graphs?

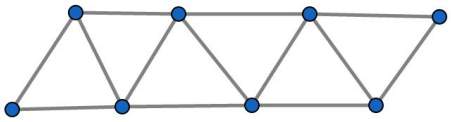
A: 

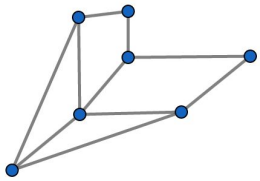
B: 

C: 

D: 

E: 

F: 

G: 

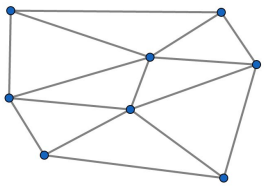
Can you make a graph with chromatic number equal to 5?

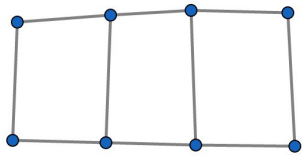
Independence number and chromatic number

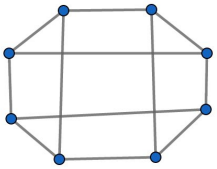
Consider as many different graphs as you can with 8 vertices. Then fill in the table below. Some sample graphs are shown below, but don't forget to add your own!

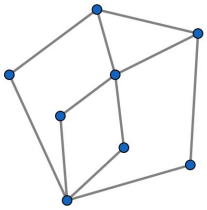
What patterns do you notice?

Graph	Maximum independent set (color)	Minimum number of colors

A: 

B: 

C: 

D: 

E: