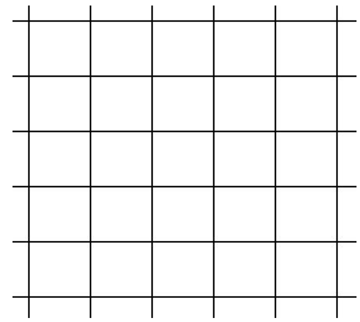
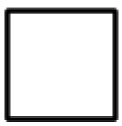
Regular Tessellations

A **regular tessellation** means covering the plane with a single regular polygon so that sides and corners line up perfectly. Regular tessellations are very useful for making designs.

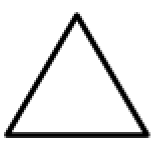
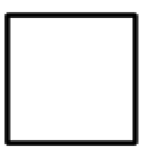
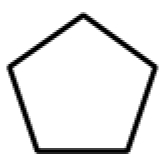
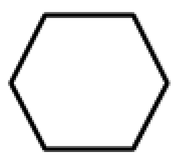
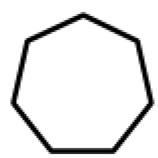
***Task**:* Which two other regular polygons can be used to make a regular tessellation? Fill in the table below to describe these three regular tessellations.



Regular Quadrilateral (Square) Tessellation

Repeating Shape	**# sides (*N*)**	**interior angle (*a*)**	**# at a corner (*C*)**
			

Task: Complete the table below to discover a formula for how ***N*** and ***a*** are related to each other.

				
***N* =**	***N* =**	***N* =**	***N* =**	***N* =**
***a* =**	***a* =**	***a* =**	***a* =**	***a* =**

Formula relating *N* and *a* :

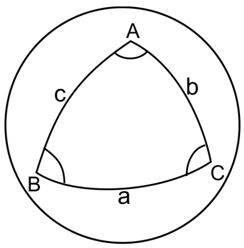
In a regular tessellation, ***aC*=360**. Use this to remove *a* from your formula, leaving just *N* and *C*.

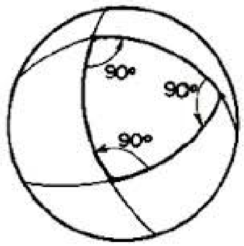
Formula relating *N* and *C* :

Regular Tessellations on a Sphere

Previously, we saw a regular tessellation of the plane has the property $(N-2)(C-2)=4$.

What happens if $(N-2)(C-2) < 4$? This could only happen if the angles of our repeating polygon are larger than they are "supposed" to be in the plane. This can happen on a sphere!





***Task**:* Use this idea to try tessellating the sphere with regular triangles.

Task: In the table, list combinations of whole numbers *N* (≥3) and *C* (≥3) where $(N-2)(C-2) < 4$. Does each combination lead to a regular tessellation of the sphere?

N	*C*	*(N-2)(C-2)*	Can this Tessellate the Sphere? (If Yes, draw it)

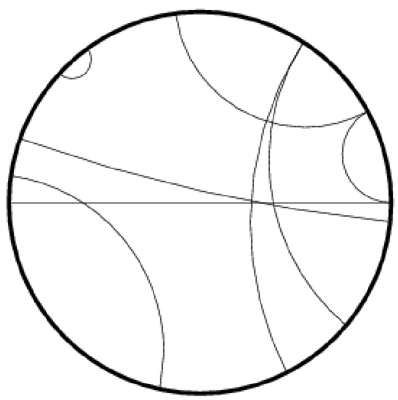
What do you notice about the shapes made by the corners of your tessellations?

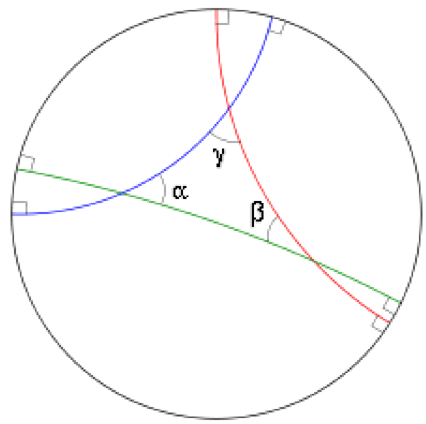
The Poincaré Disk

Is it possible to create a tessellation with $(N-2)(C-2) > 4$? This could only be if the angles of our repeating polygon are smaller than they are "supposed" to be in the plane.

Surprisingly, the answer is **yes**!

The Poincaré Disk: In this geometric model, "straight lines" are parts of circles that meet the outside circle at right angles.





The Poincaré disk is like a sphere because it is curved. The Poincaré disk is unlike the sphere because we can't hold it in our hands. Instead we can only work with "maps" of it.

You can play with the Poincaré disk at malinc.se/math/noneuclidean/poincarediscen.php and cs.unm.edu/~joel/NonEuclid/NonEuclid.html

Task*: Can you make a square in the Poincaré disk?*

As we said the Poincaré disk is like a map. Remember that maps have distortion (e.g., Antarctica isn't nearly as large as it looks on many maps).

The hyperbolic circle tool shows you what points are equidistant to a given "center".

Task*: What kind of distortion is happening in the Poincaré disk? Compare distances between points in the middle of the disk versus the edge.*

Regular Tessellations of the Poincaré Disk

You can explore possibilities for tessellating the Poincaré Disk at
malinc.se/math/noneuclidean/poincaretilingen.php

Task: In the table, list combinations of whole numbers *N* (≥3) and *C* (≥3) where $(N-2)(C-2) > 4$. Does each combination lead to a regular tessellation of the Poincaré Disk?

N	*C*	*(N-2)(C-2)*	Can this tessellate Poincare's Disk? (If Yes, draw it)

Escher used these tilings to make "vanishing tessellation" art. You can make your own vanishing tessellation art at
malinc.se/m/ImageTiling.php