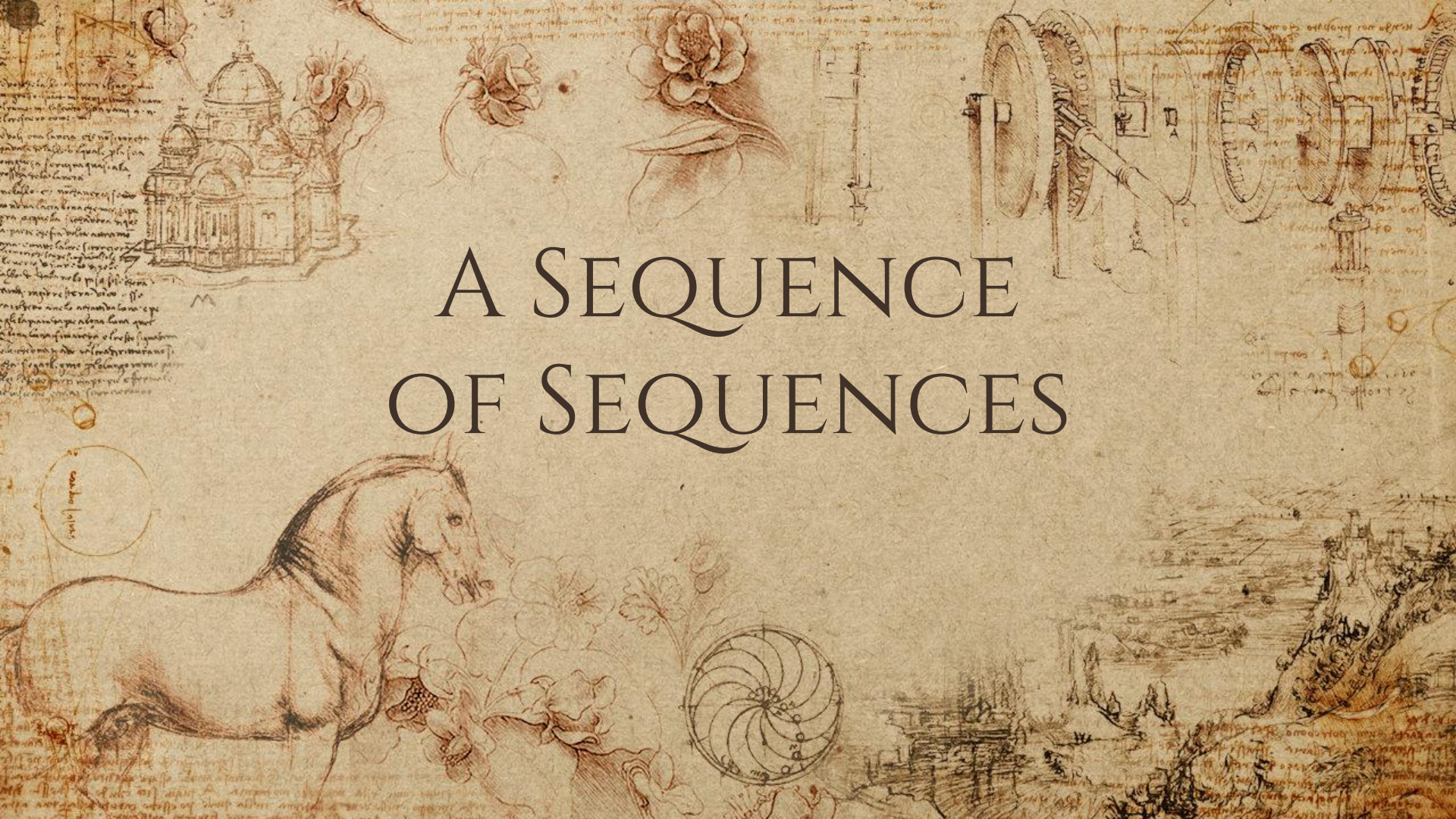
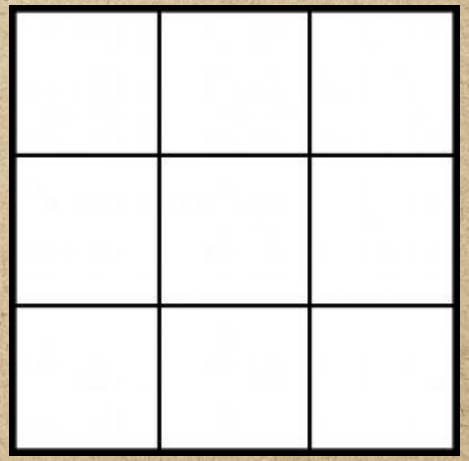


# A Sequence of Sequences

# Magic Squares



Fill in the square using the numbers 1-9 (only once), such that the sum of each row, column, and major diagonal is the same.

# Magic Constants

The sum of each row, column, and major diagonal for an *N X N* magic square is called the magic constant.

What is the magic constant for the 3 x 3 magic square?

What is the magic constant for the 4 x 4 magic square?

What is the magic constant for the 5 x 5 magic square?

# Magic Constants

What is the magic constant for the *N* x *N* magic square?

# Magic Constants - Proof

The sum of the first *N* integers is given by the formula

$$N(N+1)/2$$

Since the *NTH* magic square contains the first $N^2$ integers, the sum of all the numbers in the *NTH* magic square is

$$N^2(N^2+1)/2$$

# Magic Constants - Proof

This sum is split into $N$ different rows (or columns), so the sum of each row is

$$N^2(N^2+1)/2N$$

*OR*

$$N(N^2+1)/2$$

# Sequence of Sequences

## Magic Constants

*1, 5, 15, 34, 65, 111, 175, 260, 369, 505, 671, 870, 1105,...*

# Lazy Caterer

What is the maximum number of pieces a pizza can be cut into with one straight cut?

What is the maximum number of pieces a pizza can be cut into with two straight cuts?

What is the maximum number of pieces a pizza can be cut into with three straight cuts?

# Lazy Caterer

What is the maximum number of pieces a pizza can be cut into with *N* straight cuts?

# Lazy Caterer - Proof

**Proof #1:**

Sequence = 1, 2, 4, 7, 11, ....

First Difference = 1, 2, 3, 4, ...

Second difference = 1, 1, 1, 1, ...

Thus, we know the formula is a 2nd degree equation.

# Lazy Caterer - Proof

$$F(N)=AN^2+BN+C$$

$F(0) = A(0)^2+B(0)+C = 1$ So, $C = 1$

Now,

$F(1) = A(1)^2+B(1)+1 = 2$ and $F(2) = A(2)^2+B(2)+1 = 4$

We get the system,

$$A + B = 1$$
$$4A + 2B = 3$$

# Lazy Caterer - Proof

Solving the system gives us

$$A = \frac{1}{2} \text{ and } B = \frac{1}{2}$$

Thus,

$$F(N) = \frac{1}{2} N^2 + \frac{1}{2} N + 1$$

Or

$$F(N) = (N^2 + N + 2)/2$$

# Lazy Caterer - Proof

**Proof #2:**

The *N*th line must cross each previous line, cutting the *N*th line into *N* segments.

Each of the *N* segments divides an existing region into two regions. This adds *N* new regions to the previous solution.

This gives us: $F(N) = N + F(N-1)$

# Lazy Caterer - Proof

We can work backwards using this recursive formula getting

$$F(N) = N + N-1 + N-2 + N-3 + .... + 1 + F(0)$$

This can be rewritten as

$$F(N) = (1 + 2 + 3 + ... + N) + 1$$

$$= (N^2 + N) /2 = 1$$

$$= (N^2 + N + 2)/2$$

# Sequence of Sequences

## Magic Constants

*1, 5, 15, 34, 65, 111, 175, 260, 369, 505, 671, 870, 1105,...*

## Lazy caterer

*1, 2, 4, 7, 11, 16, 22, 29, 37, 46, 56, 67, 79, 92, 106, ...*

# Lazy caterer II

What is the maximum number of pieces a donut can be cut with one straight vertical cut?

What is the maximum number of pieces a donut can be cut with two straight vertical cuts?

What is the maximum number of pieces a donut can be cut with three straight vertical cuts?

# Lazy caterer II

What is the maximum number of pieces a donut can be cut with *n* straight vertical cuts?

# Lazy caterer II - Proof

Sequence: 2, 5, 9, 14, ...

First Difference: 3, 4, 5, ...

Second Difference: 1, 1, 1, ...

Thus, we know the formula is a 2nd degree polynomial.

# Lazy caterer II - Proof

$$F(N) = AN^2 + BN + C$$

## Solve the system of equations:

$$F(1) = A(1)^2 + B(1) + C = 2$$
$$F(2) = A(2)^2 + B(2) + C = 5$$
$$F(3) = A(3)^2 + B(3) + C = 9$$

## Solution

$$F(N) = N(N+3)/2$$

# Sequence of Sequences

## Magic Constants

*1, 5, 15, 34, 65, 111, 175, 260, 369, 505, 671, 870, 1105,...*

## Lazy caterer

*1, 2, 4, 7, 11, 16, 22, 29, 37, 46, 56, 67, 79, 92, 106, ...*

## Lazy caterer II

*2, 5, 9, 14, 20, 27, 35, 44, 54, 56, 77, 90, 104...*

# Triangles!!!

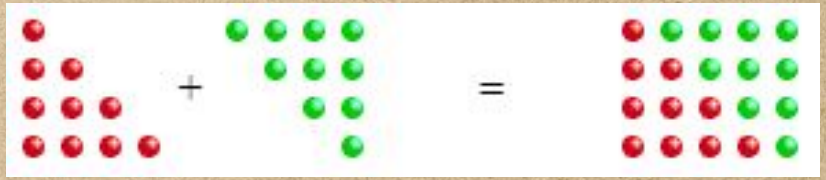
What are the triangle numbers?

1, 3, 6, 10, 15, 21, .....

What is the *n*th triangle number?

# Triangles!!! - Proof

Each triangle can be doubled and paired with itself to form a *N* by *N+1* rectangle.



There are *N(N+1)* objects in each rectangle.
Therefore, each triangle contains *N(N+1)/2* objects.

$$F(N)=N(N+1)/2$$

# Sequence of Sequences

## Magic Constants

*1, 5, 15, 34, 65, 111, 175, 260, 369, 505, 671, 870, 1105,...*

## Lazy caterer

*1, 2, 4, 7, 11, 16, 22, 29, 37, 46, 56, 67, 79, 92, 106, ...*

## Lazy caterer II

*2, 5, 9, 14, 20, 27, 35, 44, 54, 56, 77, 90, 104...*

## Triangle Numbers

*0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, ...*

# Floyd's Triangle

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | |
| 2 | 3 | | | | | | | |
| 4 | 5 | 6 | | | | | | |
| 7 | 8 | 9 | 10 | | | | | |
| 11 | 12 | 13 | 14 | 15 | | | | |
| 16 | 17 | 18 | 19 | 20 | 21 | | | |
| 22 | 23 | 24 | 25 | 26 | 27 | 28 | | |
| 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | |
| 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 |