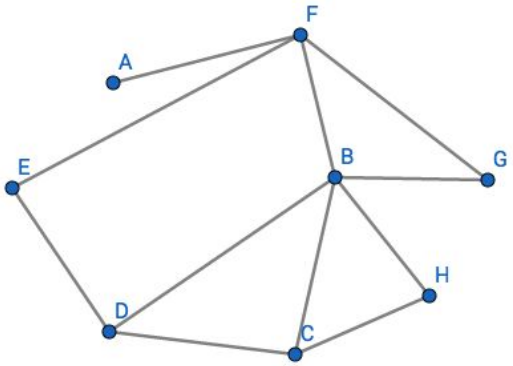
Graphs

Boise Math Circle



A **graph** is a diagram with dots, called **vertices** (singular: **vertex**), and lines joining the dots, called **edges**. For example the graph to the right has ___ vertices and ___ edges.

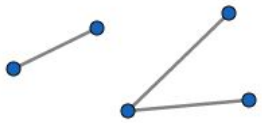
Graphs are useful because they can store and display relational information such as electrical networks, molecular bonds, internet links, personal relationships, geographic connections such as roads or bridges or airline routes, etc.

There are a number of keywords that we can use to talk about a graph. For starters, two vertices are called **adjacent** if they are joined by an edge. Then for any vertex, the **degree** of the vertex is the number of vertices that are adjacent to it. The graph above has ___ vertices of degree 1, ___ of degree 2, ___ of degree 3, ___ of degree 4, and ___ of degree 5.

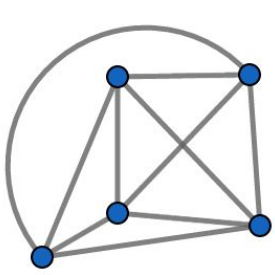
A **walk** in a graph is a list of vertices, subject to the rule that any two vertices which are adjacent in the list are adjacent in the graph too. A walk can begin at any vertex. An example of a walk in the graph above would be _,_,_,_,_,_,_.

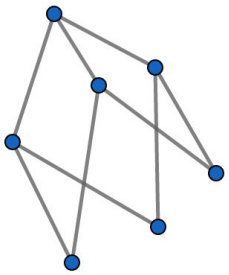
A **closed walk** is a walk that starts and ends at the same vertex. An example of a closed walk in the graph above is G, B, , , B, G.

A graph is **connected** if there is a walk from any vertex to any other vertex. Otherwise the graph is **disconnected**. For example, the graph above is connected, but the graph at right is disconnected.



A graph is **planar** if it can be drawn on paper without the edges crossing one another. The first graph below is not planar, and the second graph below is planar. Can you redraw the second graph to show why it is planar? Can you explain why the first graph can never be drawn without edges crossing?





Questions for consideration:

- If a graph has N vertices, how many edges can it have, without duplicating an edge?
- If a graph has N vertices, how few edges can it have, and still be connected?
- Is it possible for a graph to have no cycles (other than cycles consisting of just one vertex)?
- When are two graphs equivalent to each other?