Cantor's Pairing Function

Cantor's pairing function F is a function with two natural number inputs m and n and which assigns outputs according to the rule:

$$F(m,n) = \tfrac{1}{2}(m+n)(m+n+1)+n$$

Example: To compute $F(2,3)$, use $m=2$, $n=3$, so that:

$$F(2,3) = \tfrac{1}{2}(2+3)(2+3+1)+3$$

$$F(2,3) = \boxed{}$$

Task 1:

Calculate the values of F below.

- $F(1,2)$
- $F(2,1)$
- $F(0,5)$
- $F(3,3)$
- $F(10,20)$

Task 2:

Let's start to look for patterns in the values of F.

- $F(1,0)$
- $F(2,0)$
- $F(3,0)$
- $F(4,0)$
- $F(5,0)$
- What is the pattern?

Task 3:

- What **inputs** m and n result in a value $F(m,n)=9$?
- What inputs m and n result in a value $F(m,n)=17$?
- What inputs m and n result in a value $F(m,n)=100$? Can you be sure there is an answer? Can you be sure there is only one answer?

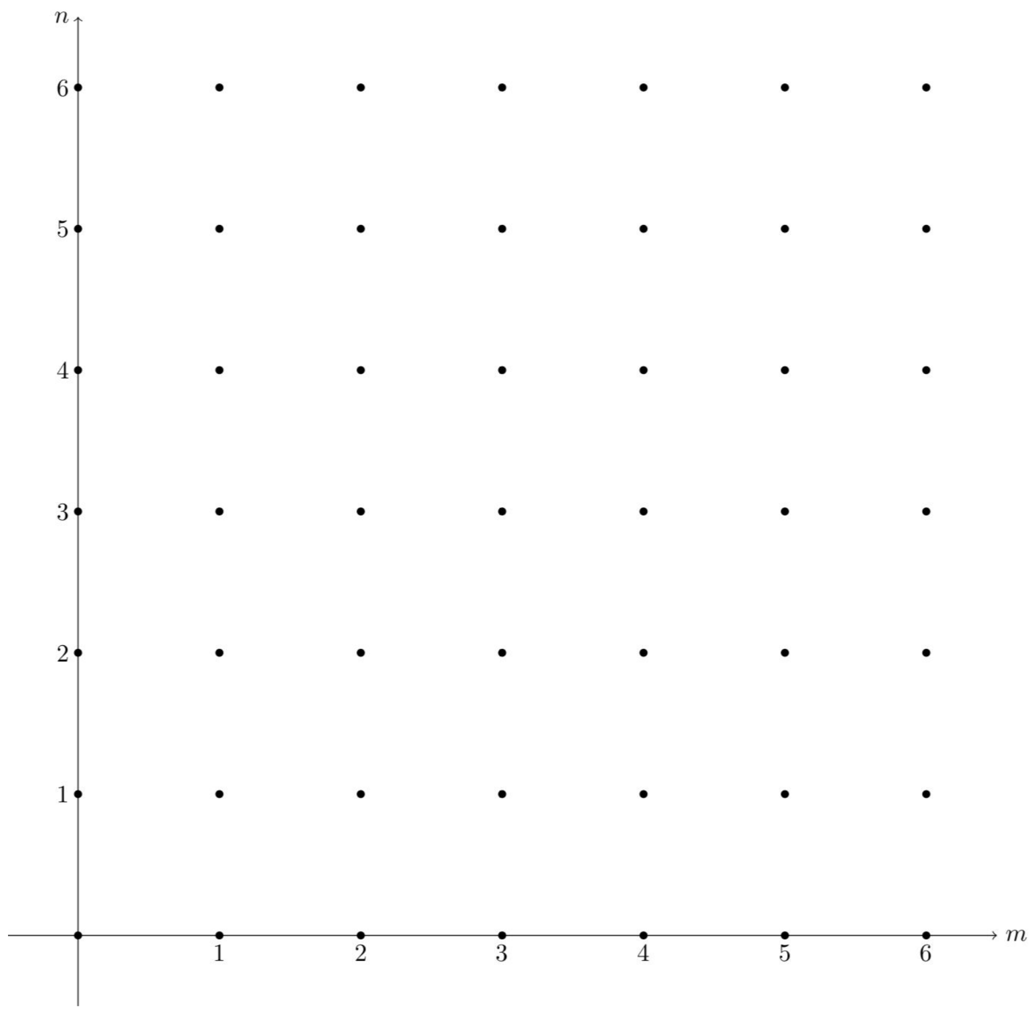
Cantor’s Pairing Pattern

Cantor's Pairing Pattern

The goal of this worksheet is to visualize Cantor's pairing function using the **integer lattice**.

Task

At each lattice point at coordinate (m, n), write the corresponding value of the Cantor pairing function $F(m, n)$. There are 49 dots, so you may want to split up the work with your partners!



What patterns do you notice?

Questions for further discussion

- We have said that the domain of the Cantor pairing function consists of all pairs of natural numbers. What is the rangef of the Cantor pairing function?
- Is the Cantor pairing function a bijection between its domain and range? If it were, would this surprise you? Why?
- What if we consider larger domains, such as integers or real numbers? Can the Cantor pairing function be a bijection between the larger domain and some other set?