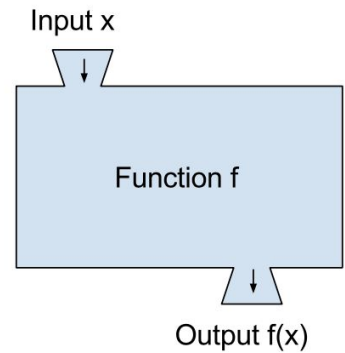


# Functions

A **function** is a math gadget that has inputs and outputs, and for each input, it creates one output. We often think of a function as a machine which has a conveyor belt coming in (the inputs) and a conveyor belt going out (the output).



If  $f$  is a function, and  $x$  is an input, then the output is labeled  $f(x)$ .

*Example:* If  $f$  is the function which squares its input, then we write:

$$f(3) = 9 \qquad f(5) = 25 \qquad f(-2) = 4 \qquad f(2.5) = 6.25 \qquad f(x) = x^2$$

Many functions are built into calculators, such as  $x^2$ ,  $\sqrt{x}$ ,  $e^x$ ,  $\sin(x)$ , and so on.

The **domain** of a function  $f$  consists of all allowed inputs  $x$ . The **range** of  $f$  consists of all possible outputs  $f(x)$ .

*Example:* For example if  $f$  is the square-root function  $f(x) = \sqrt{x}$  then we could take its domain to be all nonnegative numbers. We typically don't allow the domain to be all real numbers, since negative numbers have no square root (unless we agree to allow complex numbers). The range would also be the nonnegative real numbers.

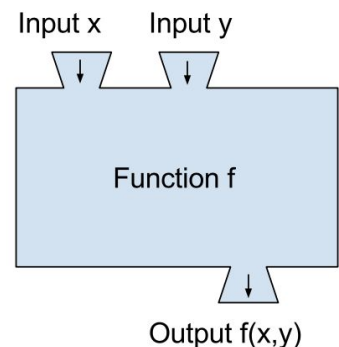
A function  $f$  is called a **bijection** or a **one-to-one correspondence** between its domain and range if distinct inputs always correspond to distinct outputs.

*Example:* Define  $f(n) = 2n$ , with the domain all natural numbers. Then the range of  $f$  consists of all even natural numbers. Since distinct  $m, n$  always correspond to distinct values  $2m, 2n$ ,  $f$  is a bijection between the natural numbers and the even natural numbers.

When there is a bijection between two sets, we say they have the **same size**.  
So the set of all natural numbers is the same size as the set of all even numbers!

## Multivariate Functions

A function can also have **more than one input** at a time. If there are two inputs, we think of our function as having two conveyor belts coming in, but still only one conveyor belt going out.



If  $f$  is a function with two inputs  $x$  and  $y$ , then the corresponding output is written as  $f(x,y)$ .

*Example:* Addition and multiplication are functions with two inputs.

$$\begin{array}{lll} A(x,y) = x + y & A(3,9) = 12 & A(10,0) = 10 \\ M(x,y) = x \cdot y & M(3,9) = 27 & M(10,0) = 0 \end{array}$$