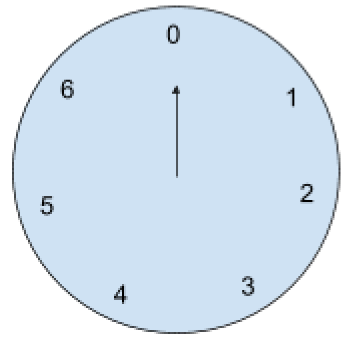
Modular counting



Consider the spinner shown at right. As the arrow advances clockwise around the circle, it will pass the numbers 0, 1, 2, 3, 4, 5, 6 and then it will start repeating. In **modular counting** are given a number N where the counting resets to 0. In this picture, N is 7 (one more than the largest number shown).

After N resets to 0, subsequent numbers are equivalent to earlier numbers: N+1 is equivalent to 1, N+2 to 2, and so on. If two numbers are equivalent in this fashion, we say they are **congruent**, which is written with the symbol $a \equiv b$ (mod N). Here are some example congruences that are true when N is 7:

$1 \equiv 8$ (mod 7) $12 \equiv 26$ (mod 7) $78 \equiv 1$ (mod 7)

It works for negative numbers too:

$2 \equiv -5$ (mod 7) $-21 \equiv 0$ (mod 7) $-100 \equiv -30$ (mod 7)

The rule for a congruence is that the difference of the two numbers is divisible by N, or equivalently, both numbers have the same remainder when divided by N.

Modular counting can help us answer a question like:

- What time of day will it be in 1000 hours?

To solve this question we use a modulus N=24, that is, a clock with numbers 0, ..., 23. According to this clock, 24 is equivalent to 0. By the same reasoning, 240 is equivalent to 0. In fact the highest number under 1000 that would be equivalent to zero is 984. This means in 1000 hours, it will be the same time as it will be in just 16 hours. In symbols we write:

$$1000 = 41 \times 24 + 16 \equiv 16 \text{ (mod 24)}$$

In 1000 hours it will also be the same time of day as it was 8 hours ago!

Some further questions that will help you explore patterns in modular counting.

- When are two numbers congruent mod 2?
- When are two numbers conruent mod 10?
- What is the last digit of $9 \times 1234567891234567$?
- What time will it be in 3^{50} hours?
- What is the last digit of 7^{100}?